

## A test of higher order electroweak theory in $Z^0$ decays to two leptons with an associated pair of charged particles

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The OPAL detector at LEP was used to study the reactions  $e^+e^- \rightarrow e^+e^-V$ ,  $e^+e^- \rightarrow \mu^+\mu^-V$  and  $e^+e^- \rightarrow \tau^+\tau^-V$ , where V represents an additional pair of oppositely charged tracks coming directly from the event vertex. Rates for these processes were determined using selection criteria similar to those described in a recent publication by the ALEPH Collaboration in which a possible excess in the  $\tau^+\tau^-V$  channel was reported. In the OPAL data no discrepancy was found between the measured production rates and those expected from known physics processes in any of the three channels. The measured distributions of the invariant mass of the V and angle between the V and closest lepton are also described adequately within the context of the standard model.

## 1. Introduction

This letter reports on a study of  $\ell^+\ell^-V$  events produced in  $e^+e^-$  collisions using the OPAL detector at LEP. In this context,  $\ell^+\ell^-$  represents the primary electron, muon or tau pair in a leptonic decay of the  $Z^0$  and the  $V$  is an additional pair of oppositely charged particles, leptonic or hadronic, coming directly from the event vertex. Within the standard model such events are expected to occur as a result of pair production by virtual initial or final state bremsstrahlung photons and the production rates can be reliably calculated. Current precision measurements of the leptonic branching fractions of the  $Z^0$  [1,2] do not explicitly take into account the  $\ell^+\ell^-V$  events which are expected to contribute at the 0.1% level. Because events from the related process,  $Z^0 \rightarrow q\bar{q}V$ , tend to be included in the  $Z^0$  hadronic branching fraction measurements it is evident that as the measurement of the ratio of leptonic to hadronic branching fractions of the  $Z^0$  approaches the 0.1% level, it will be necessary to account for the  $\ell^+\ell^-V$  events in the selection of  $Z^0 \rightarrow \ell^+\ell^-$  events. In addition the  $\ell^+\ell^-V$  and  $q\bar{q}V$  processes are important backgrounds in searches for a Higgs boson at LEP [3]. For these two reasons it is important to verify the validity of the currently available calculations. Although studies of the processes  $e^+e^- \rightarrow e^+e^-e^+e^-$  and  $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$  at the PEP and PETRA colliders have shown no discrepancies with QED [4], the AMY Collaboration at the TRISTAN collider has reported an observation of an anomalously high  $e^+e^-\mu^+\mu^-$  production rate [5]. Furthermore, in a recent publication by the ALEPH

Collaboration on  $\ell^+\ell^-V$  production at LEP, a possible excess of events in the  $\tau^+\tau^-V$  channel is reported [6]. In order to search for an anomalous  $\tau^+\tau^-V$  production rate as reported by ALEPH, the present OPAL analysis parallels the ALEPH analysis as closely as possible, given the differences between the OPAL and ALEPH detectors. The analysis also addresses the issue of a possible anomaly in the  $e^+e^-\mu^+\mu^-$  channel.

The OPAL detector is described in detail in ref. [7]. It has a central detector consisting of a vertex chamber, a 'jet' chamber, and  $z$ -chambers inside a solenoidal magnetic field. The coordinate system is defined with  $+z$  along the  $e^-$  beam,  $\theta$  and  $\phi$  being the polar and azimuthal angles, respectively. The magnet coil is surrounded by a time-of-flight counter array, a lead-glass electromagnetic calorimeter with a presampler, and an instrumented magnet return yoke serving as a hadron calorimeter which in turn is surrounded by a set of muon chambers. The endcap system occupies the region  $|\cos\theta| \gtrsim 0.8$ . It consists of electromagnetic and hadron calorimeters, muon chambers and a low-angle forward detector which is used to determine the luminosity to an accuracy of better than 1% [1]. The analysis presented here used data collected at centre-of-mass energies between 88.2 and 94.2 GeV which amounted to an integrated luminosity of  $18.6 \text{ pb}^{-1}$  and corresponded to 429 000 multihadronic decays of the  $Z^0$ .

## 2. Simulation of known physics processes

In order to simulate the various processes which could contribute to the  $\ell^+\ell^-V$  signal, several Monte Carlo data sets were used. The response of the OPAL detector to the generated particles in each case was modelled fully using a simulation program [8] based on the GEANT [9] package. In all cases, the Monte Carlo and real data were analysed in an identical manner.

The four-fermion Monte Carlo [10] simulates to lowest order the purely leptonic processes  $ee \rightarrow eeee$ ,  $ee\mu\mu$ ,  $ee\tau\tau$ ,  $\mu\mu\mu\mu$ ,  $\mu\mu\tau\tau$ ,  $\tau\tau\tau\tau$ , and is the same generator as was used in ref. [6]. Data sets corresponding to integrated luminosities of several hundred inverse picobarns per process were generated at various energies distributed over the  $Z^0$  peak in accord with the LEP energy scans. The cross sections ob-

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tained from the generator were corrected in order to obtain predicted cross sections valid in the improved Born approximation for centre-of-mass energies near the  $Z^0$  mass. The generator does not include initial or final state radiation effects, apart from the pair production by the virtual photon. Initial state radiation (ISR) is expected to have an important influence close to the  $Z^0$  resonance and a correction factor was applied to the predicted rates to account for ISR effects. A correction factor, given by  $\sigma(Z^0 \rightarrow q\bar{q} \text{ with ISR})/\sigma(Z^0 \rightarrow q\bar{q} \text{ without ISR})$ , was computed at each centre-of-mass energy point using the Monte Carlo described in ref. [11]. This factor ranges between 0.74 and 1.47.

The four-fermion Monte Carlo was also used to determine  $\ell^+\ell^-V$  production rates for the case in which the  $V$  is a pair of charged hadrons. Assuming that lepton pairs of the four-lepton final state come from a single virtual photon emission, the rate for  $q\bar{q}$  emission was estimated to be the rate for  $\mu^+\mu^-$  emission scaled by the  $R$ -value,

$$\frac{\sigma(e^+e^- \rightarrow \text{two charged hadrons} + \text{anything})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)},$$

as measured in lower energy  $e^+e^-$  annihilation experiments [12] and evaluated at an energy equal to the invariant mass of the  $V$ . This treatment included contributions from  $\pi^+\pi^-\pi^0$  states, which were not explicitly removed in the analysis. The selection efficiency for these states is difficult to evaluate. As a cross-check, therefore, the weights were also determined using the data on the time-like pion form factor [12] which includes contributions only from the  $\pi^+\pi^-$  final state. An overall systematic error of 15% was assigned to the predicted event rates from the four-fermion Monte Carlo in order to cover uncertainties associated with the lack of initial and final state radiation effects in the generator (10% and 5% respectively) and the  $R$ -value weighting (10%).

In order to evaluate the background coming from the multihadronic decays of the  $Z^0$ , the JETSET Monte Carlo [13] was used with the parameters tuned to fit the global event shape distributions in OPAL multihadron data [14]. A total of 432 000 events was generated and passed through the full detector simulation and reconstruction code. As a cross check on potential fragmentation systematics, a sample of 96 000 events generated using the HER-

WIG Monte Carlo [15] with parameters tuned to the OPAL data [14] was also used.

Backgrounds from radiative dimuon and Bhabha events in which the photon converted in the material of the detector were estimated using 70 000 events generated with the KORALZ Monte Carlo program [16] and 74 000 events generated with the BABAMC Monte Carlo program [17], respectively. The integrated luminosities of these samples were  $62 \text{ pb}^{-1}$  and  $23.0 \text{ pb}^{-1}$ , respectively, after the scan over the  $Z^0$  resonance was taken into account. Background from  $\tau$  pair production was evaluated using 122 000 KORALZ Monte Carlo events. This corresponds to a scan-equivalent integrated luminosity of  $110 \text{ pb}^{-1}$ .

Non-resonant  $t$ -channel two-photon processes were simulated with the generator described in ref. [18]. Monte Carlo event samples were generated for each of the various processes of interest with integrated luminosities of  $20 \text{ pb}^{-1}$ .

### 3. Event selection and analysis

In order to obtain a clean sample of  $\ell^+\ell^-V$  events, it was necessary to reject various sources of background including  $Z^0 \rightarrow \tau^+\tau^-$  events, multihadron events, radiative di-lepton events (in which the photon converted in the material of the detector) and two-photon events.

For the purposes of this analysis three mutually exclusive classes of tracks have been defined: "good quality tracks", "medium quality tracks" and "conversion tracks". Two oppositely charged tracks were identified as coming from the conversion of a photon and were referred to as *conversion tracks* if (i) they each had more than 12 jet chamber hits (ii)  $\Delta_{xy} < 4 \text{ mm}$ , where  $\Delta_{xy}$  was the minimum distance between the points where tangents to the tracks at those points, projected onto the  $xy$  plane, were parallel, and (iii) the reconstructed mass of the track pair, assuming zero particle mass, was less than 150 MeV. A track which has not been identified as originating from a conversion was classified as a *good quality track* if (i)  $p_t > 100 \text{ MeV}$  (where  $p_t$  is the transverse momentum of the track relative to the beam axis), (ii) the number of jet chamber hits on the track was at least 20 and more than 50% of the geometrically possible number

of hits for its polar angle, (iii) the distances of closest approach to the origin in the plane perpendicular to the beam ( $|d_0|$ ) and parallel to the beam ( $|z_0|$ ) were less than 1 cm and 50 cm, respectively, and (iv)  $|\cos \theta| < 0.95$ . A *medium quality track* was defined as a non-conversion track which failed the good quality criteria but had (i)  $p_t > 100$  MeV, (ii) the number of hits greater than 20, and (iii)  $|d_0| < 1$  cm if  $p_t < 250$  MeV. (Often very low momentum tracks were segmented into several tracks in the reconstruction phase. This last requirement ensured that these low momentum tracks were counted only once by taking advantage of the fact that these excess segments usually had large reconstructed  $d_0$  values because of the large amount of multiple scattering.)

An event was selected if it had either four or six good quality tracks and if the total energy as determined from the momenta of these tracks was greater than 16 GeV. This energy requirement efficiently removed two-photon events from the sample. Any six-prong event was rejected if there were, in addition to the good tracks, any medium quality tracks or conversion tracks. Four-prong events were removed at this stage only if they contained more than two medium quality tracks or more than two conversion tracks. This allows the analysis to retain sensitivity to four-prong  $\ell^+\ell^-V$  events which contained a conversion. Only good quality tracks were used during the rest of the analysis. A total of 3783 events survived the above cuts, consistent with a predicted total of 3755 events in the Monte Carlo simulation (see table 1).

As is apparent from table 1, the major background at this stage of the analysis was due to  $e^+e^- \rightarrow \tau^+\tau^-$  events. In order to remove this background it was required that, in four-prong events, no combination of three tracks (triplet) having a charge of  $\pm 1$  could have a mass less than 3 GeV. In all mass calculations a zero particle mass was assumed. In six-prong events, masses were calculated for all possible combinations of three tracks having a charge of  $\pm 1$ . A six-prong event was required to have exactly one such combination of three tracks with a mass less than 2 GeV. The momentum vector sum of the triplet having a mass less than 2 GeV was identified as a  $\tau$  momentum vector and was treated as a single particle in the rest of the analysis. This process ensured that in a six-prong event the three-prong  $\tau$  candidate was identified unambiguously. The mass of the remaining three tracks

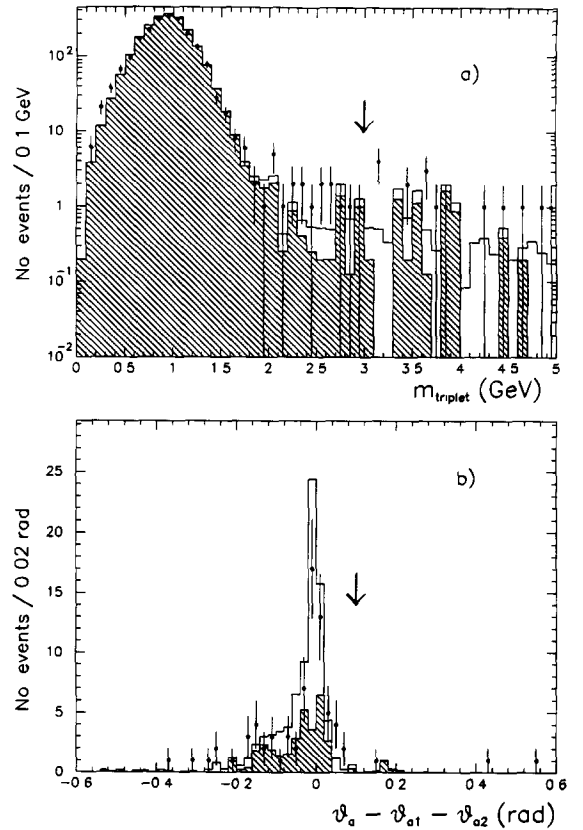


Fig. 1 (a) Distribution of the minimum triplet mass in four-prong events and the mass of the three tracks in six-prong events that are not identified as the three-prong tau. Distributions for both the data (points with error bars) and for Monte Carlo expectations (line histograms) are presented after all cuts except the cut on the triplet mass. (b) The distributions of  $\theta_a - \theta_{a1} - \theta_{a2}$  for data and Monte Carlo are shown for the combination of tracks having the smallest value for this quantity. The plot is presented after all cuts were applied except this one and those on lepton pair classification. In both (a) and (b) the shaded part of the histogram represents the contributions from all sources other than  $\ell^+\ell^-V$ . The arrows in the figures indicate the position of the cuts.

in the six-prong event was required to be greater than 3 GeV. The distribution of the smallest triplet mass in the four-prong events and the mass of the three tracks not identified as the three-prong  $\tau$  in the six-prong events is shown in fig. 1a after all other cuts were applied. Distributions for both the data and for the Monte Carlo expectations are presented. The number of events surviving the triplet mass cut was 202.

Table 1

The expected and observed numbers of events. A summary of the comparison between data and the Monte Carlo simulation of all known processes as each successive requirement is imposed. The Monte Carlo is normalised to the integrated luminosity of the data. The expected numbers of  $\ell^+\ell^-V$  events are given in the fourth column from the left and are followed by the prediction of the multihadron background from JETSET and the backgrounds from tau pairs, muon pairs, Bhabhas and two-photon processes. The errors in the Monte Carlo total column include contributions from Monte Carlo statistics and the 15% uncertainty on the  $\ell^+\ell^-V$  rate discussed in the text.

Cut description	Data	Monte Carlo						
		total	$\ell^+\ell^-V$	$q\bar{q}$	$\tau\tau$	$\mu\mu$	$ee$	$\gamma\gamma$
charged multiplicity	3783	3755 $\pm 30$	67.1	146	3476	5.2	56.0	4.6
conversion removal								
charged energy								
triplet mass	202	178 $\pm 12$	59.0	56	42.8	3.7	15.5	1.9
opening angle V	76	88.2 $\pm 9.6$	53.2	15	6.3	2.6	10.4	0.9
and opening angle $\ell^+\ell^-$								
recoil kinematics	70	81.3 $\pm 9.2$	51.3	12	5.2	2.6	10.4	0
$\ell^+\ell^-$ identification	50	57.1 $\pm 5.5$	44.0	0	2.9	1.4	8.7	0
$e^+e^-V$	21	32.3 $\pm 4.7$	23.6	0	0	0	8.7	0
$\mu^+\mu^-V$	18	15.3 $\pm 2.2$	13.8	0	0	1.4	0	0
$\tau^+\tau^-V$	11	9.5 $\pm 1.8$	6.6	0	2.9	0	0	0

compared to 178 events expected in the Monte Carlo simulation (see table 1). Note that the ALEPH Collaboration used a triplet mass cut of 1.7 GeV instead of 3 GeV. This point will be discussed in more detail below.

Within the events which survived these cuts, V candidates were formed from the various combinations of oppositely charged tracks with the remaining tracks considered as lepton pair candidate tracks. The opening angle between the two lepton candidates was required to be greater than  $90^\circ$  and the opening angle between the two tracks forming the V was required to be less than  $120^\circ$ . These cuts are identical to those used by the ALEPH Collaboration and help to reduce multihadron and  $\tau^+\tau^-$  background. After these cuts a total of 76 events remained, to be compared with the expectation of 88.2 events from the Monte Carlo analysis. The Monte Carlo expectation comprised 53.6  $\ell^+\ell^-V$  events, 14.9 multihadron events and 6.3  $Z^0 \rightarrow \tau^+\tau^-$  decays, together with contributions from  $Z^0 \rightarrow \mu^+\mu^- (\gamma)$ ,  $Z^0 \rightarrow e^+e^- (\gamma)$  and two-photon processes. Note that at this stage, and subsequently, cuts were applied to  $\ell^+\ell^-V$  configurations rather than to events. An event was retained if any configuration within the event passed the cuts. Also,

for the selection criteria described below, the electromagnetic energy was used instead of the track energy if the electromagnetic energy associated to a track was larger than the track energy. This retained sensitivity to electrons which radiated.

In the ALEPH analysis [6] a series of cuts was made after the lepton pair candidates were boosted into the frame recoiling against the V. In order to provide as close a comparison as possible with the ALEPH results, the same set of cuts has been applied here:

(i)  $X_1 > 0.05$  and  $X_2 > 0.05$  where  $X_{1(2)}$  is the ratio of the lepton candidate energy to the energy kinematically available to the lepton.

(ii) The acollinearity angle between the two lepton candidate tracks ( $\theta_a$ ) was required to satisfy  $\theta_a < \theta_{a1} + \theta_{a2} + 100$  mrad. Here  $\theta_{a1(2)}$  is the maximum angle which an electron from a  $\tau$  decay can make with the  $\tau$  flight direction, given the momentum of the electron. The 100 mrad allows for the experimental resolution of the OPAL detector. The distribution of  $\theta_a - (\theta_{a1} + \theta_{a2})$  is shown in fig. 1b for the combination of tracks with the smallest value of the quantity before this cut was applied. It is evident that this distribution is well reproduced by the Monte Carlo. The corresponding cut used by the ALEPH Collaboration

was 35 mrad ( $2^\circ$ ). If a lepton candidate failed the  $X_{1(2)} > 0.05$  cut, the  $X_{1(2)}$  value was re-evaluated with the momentum of the lepton candidate redefined as the vector sum of the lepton candidate track momentum and energy vectors of the electromagnetic clusters within  $18^\circ$  of the track but not associated to the track. The configuration was rejected only if these re-defined  $X_{1(2)}$  values also failed the  $X_{1(2)} > 0.05$  cut. This algorithm increased the efficiency for selecting  $\tau^+\tau^-V$  events in which the decay products of one or both  $\tau$ 's included  $\pi^0$ 's. A total of 70 events remained after this cut was applied, to be compared with the Monte Carlo expectation of 81.3 events.

Finally, a set of cuts was applied in order to classify the lepton pair candidates as an  $e^+e^-$ ,  $\mu^+\mu^-$  or  $\tau^+\tau^-$  pair. A lepton pair candidate was considered an  $e^+e^-$  or  $\mu^+\mu^-$  pair if  $X_1 > 0.8$  and  $X_2 > 0.8$ , where  $X_1$  and  $X_2$  were defined in the recoil frame discussed above. The lepton pair candidate was classified as a  $\mu^+\mu^-$  pair if both lepton tracks had hits in the muon chambers or hits in the outer layers of the hadron calorimeter or deposited less than 2 GeV of energy in the electromagnetic calorimeter. Otherwise it was classified as an  $e^+e^-$  pair provided that the electromagnetic calorimeter energy associated with the lepton candidate tracks was more than 60% of  $(E_{cm} - E_V^{ch})$ , where  $E_{cm}$  is the centre-of-mass energy and  $E_V^{ch}$  is the energy of the charged tracks associated with the V. An event was classified as a  $\tau^+\tau^-V$  event if  $X_1 < 0.9$ ,  $X_2 < 0.9$ ,  $X_1 + X_2 < 1.2$  and  $E_{vis}/E_{cm} < 0.80$ , where  $E_{vis}$  is the visible energy in the event, obtained from the combined information from central tracking chambers, calorimeters and muon chambers. Fig. 2a displays the distribution of  $E_{vis}/E_{cm}$  for a control sample obtained with the above selection criteria with the exception that the triplet mass cut was reversed: events were selected if the triplet mass was less than 3 GeV. This sample was dominated by  $Z^0 \rightarrow \tau^+\tau^-$  events and, like the  $\tau^+\tau^-V$  event sample, contained a significant number of leptons. From this figure it can be seen that the Monte Carlo describes the data for this variable adequately. (In the ALEPH analysis a cut was made such that the total energy of a  $\tau^+\tau^-V$  event is less than 80 GeV.) In order to reduce the multihadron background in the  $\tau^+\tau^-V$  sample in this analysis further, an additional criterion was employed here which was not used in the ALEPH analysis: for  $\tau^+\tau^-V$  candidates it was required that the number of good qual-

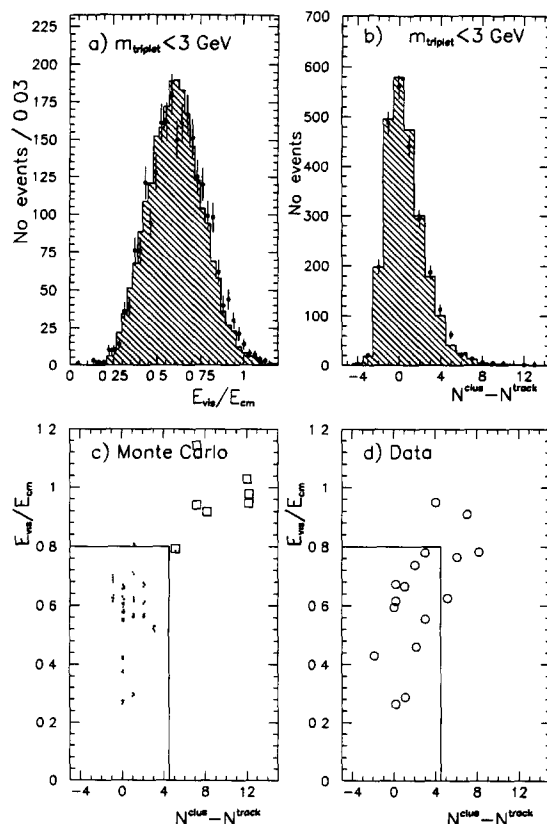


Fig. 2 (a) Scaled total energy in a control sample dominated by four-prong  $Z \rightarrow \tau^+\tau^-$  events. The data are represented by the points with error bars while the Monte Carlo is represented by the line histogram, (b) the number of electromagnetic calorimeter clusters in excess of the number of good tracks for the same control sample, (c) Monte Carlo of scaled total energy versus  $(N^{clus} - N^{track})$  for  $\tau^+\tau^-V$  events (dots) and for the multihadronic background (open squares), (d) Scaled total energy versus  $(N^{clus} - N^{track})$  in the data. In (c) and (d) the solid lines indicate the position of the cuts.

ity electromagnetic clusters in excess of the number of good quality charged tracks  $(N^{clus} - N^{track})$  be less than five. A good quality cluster in the barrel region had at least 170 MeV of deposited energy, while in the end-cap region, where the lead-glass blocks are parallel to the beam axis, a good quality cluster was required to have at least two blocks and a minimum of 250 MeV deposited energy. The distribution of  $N^{clus} - N^{track}$  for the same control sample described above is shown in fig. 2b and again there is adequate agreement between data and Monte Carlo. Scatter plots of  $E_{vis}/E_{cm}$  ver-

sus  $N^{\text{clus}} - N^{\text{track}}$  for Monte Carlo and data are shown in figs 2c and 2d, respectively. In fig 2c the dots represent  $\tau^+\tau^-V$  events while the open squares represent multihadron background events. It is evident that the predicted multihadron background was substantially reduced by the above cut while the  $\tau^+\tau^-V$  signal was expected to suffer a very small decrease.

Following the application of the above criteria, events were rejected for which no combination of tracks was classified as an  $e^+e^-V$ ,  $\mu^+\mu^-V$  or  $\tau^+\tau^-V$  candidate. In the very few cases in which more than one acceptable combination survived for an event, a unique assignment was made after all other cuts had been applied by choosing the combination with the  $V$  having the smallest invariant mass.

After these events were selected, the particles associated with the  $V$  were classified as electrons, muons or hadrons. This classification did not remove any events. The particles were considered to be an electron pair if the ratio of their energy, as measured in the electromagnetic calorimeter, to the sum of their momenta was at least 0.7. This criterion was applied to the sum and not to each particle individually in order to handle cases in which both tracks were associated to the same electromagnetic cluster because of the calorimeter's intrinsic granularity. If one of the  $V$  particles had a match to a track in the muon chambers or the outer layer of the hadron calorimeter, the particles were classified as muons. Otherwise, the  $V$  particles were assumed to be hadrons. The cross contamination was determined from the Monte Carlo and was found to be significant only for the leptonic contamination of the hadron channel where the background from electron and muon pairs being misidentified as a pair of hadrons was 23%. The backgrounds in the two leptonic channels from cross contamination was less than 10%.

#### 4. Results and discussion

The final numbers of events are summarised in table 1. A total of 50 events remained, comprising 21  $e^+e^-V$ , 18  $\mu^+\mu^-V$  and 11  $\tau^+\tau^-V$  events. Also shown in table 1 are the expected event rates from known physics processes: 32.3  $e^+e^-V$ , 15.3  $\mu^+\mu^-V$  and 9.5  $\tau^+\tau^-V$  events, including background contributions as

determined by Monte Carlo of 8.7, 1.4 and 2.9 events, respectively.

The background contamination in the  $\tau^+\tau^-V$  channel consisted of 2.9 events from  $Z^0 \rightarrow \tau^+\tau^-(\gamma)$ . The background from the multihadronic decays of the  $Z^0$  was estimated from the JETSET Monte Carlo sample which was equivalent in size to the data sample. No events survived in that sample. A cross check of the sensitivity of this background to the multihadron fragmentation scheme was performed with a smaller sample of HERWIG Monte Carlo events amounting to roughly one quarter the size of the data sample. The number of HERWIG Monte Carlo events surviving the cuts on charged multiplicity, charged energy and triplet mass was found to be  $1.6 \pm 0.3$  times larger than that for JETSET. However, no HERWIG events survived all cuts. This study was used to assign an additional systematic error of  $^{+60}_{-0}\%$  on the estimate of the multihadron background contribution at each stage in the analysis. The backgrounds were investigated further by separating the sample into four-prong and six-prong events, with the expectation that the background problems would be very different in the two sub-samples. Also, the effect of reducing the triplet mass cut from 3 GeV to 2 GeV in order to make a closer comparison with the ALEPH analysis has been examined. The results of these investigations are shown in table 2. On the basis of the data shown in table 2 and given the limited statistics in the multihadron Monte Carlo, we conclude that our six-prong  $\tau^+\tau^-V$  sample becomes heavily contaminated by multihadron background when the triplet mass cut is lowered. It is also evident that the four-prong  $\tau^+\tau^-V$  sample is not as badly contaminated. However, because of the limited Monte Carlo statistics we cannot exclude the presence of some multihadron background in the four-prong sample even when we apply a triplet mass cut of 3 GeV. The background from poorly measured  $Z^0 \rightarrow \tau^+\tau^-$  events in the four-prong sample is predicted to become increasingly important as the triplet mass cut is reduced from 3 GeV to 2 GeV. This effect appears to be larger in the data than in the Monte Carlo simulation. Thus although our Monte Carlo studies indicated that a triplet mass cut of 2.5 GeV would be sufficiently tight to reduce this background to an acceptable level, to be conservative a 3 GeV cut was applied at the cost of a relatively minor reduction in acceptance. Note, however,



Table 2

The  $\tau^+\tau^-V$  sample in the OPAL experiment for various choices of the triplet mass cut. The four-prong and six-prong samples are shown separately. The errors on the multihadron background are dominated by the Monte Carlo statistical error and a systematic error of  $^{+60}_{-0}\%$  associated with fragmentation. The errors on the  $\ell^+\ell^-V$  Monte Carlo quantities are dominated by the 15% systematic uncertainties discussed in the text. The error on the  $\tau^+\tau^-$  background is less than 25% for a triplet mass greater than approximately 2.5 GeV and arises from the uncertainties in the simulation of the detector material and Monte Carlo statistics. For triplet masses well below 2.5 GeV the uncertainty is dominated by the inadequate modelling of poorly measured  $\tau^+\tau^-$  events and is larger.

	Triplet-mass cut (GeV)	Data	Monte Carlo			
			total	$\ell^+\ell^-V$	$q\bar{q}$	$\tau\tau$
four-prong $\tau^+\tau^-V$ events	3.0	9	8.8	5.9	0	2.9
	2.5	10	11.1	6.4	1	3.8
	2.0	20	13.6	6.9	1	5.7
six-prong $\tau^+\tau^-V$ events	3.0	2	0.7	0.7	0	0.0
	2.5	2	1.7	0.7	1	0.0
	2.0	2	2.8	0.7	2	0.1

that fig. 1a shows clearly that the triplet mass resolution is modelled adequately in the Monte Carlo. This is the only cut which was changed significantly in the OPAL analysis relative to the ALEPH analysis.

In summary, the OPAL data show no indication of an excess of events in any  $\ell^+\ell^-V$  channel. In particular, the ratio of the measured  $\tau\tau V$  production rate to that predicted in the standard model is

$\tau\tau V$  production rate (data)

$\tau\tau V$  production rate (S.M.)

$$= 1.2 \pm 0.5(\text{stat}) \pm 0.2(\text{syst})$$

$< 2.3$  at the 95% confidence level

This is to be compared with an observed enhancement factor of more than four by ALEPH. A similar limit on the absolute enhancement factor has been published recently by the Mark II Collaboration based on data taken at  $\sqrt{s} = 29$  GeV [19].

The distribution of the invariant mass of the  $V$  in the OPAL analysis is presented in figs. 3a–c for  $e^+e^-V$ ,  $\mu^+\mu^-V$  and  $\tau^+\tau^-V$  events separately. The measured distributions are fully consistent with the expected distributions based on the Monte Carlo analysis, and show a tendency for the events to cluster at lower masses (0–1.0 GeV).

In figs. 4a–c the  $V$  mass is plotted for the three classifications of the  $V$ :  $e^+e^-$ ,  $\mu^+\mu^-$  and  $\pi^+\pi^-$  after the three  $\ell^+\ell^-$  channels were combined. The 50 events were classified as 23  $\ell^+\ell^-e^+e^-$ , nine  $\ell^+\ell^-\mu^+\mu^-$  and

Table 3

The expected and observed numbers of events for each classification of the  $\ell^+\ell^-$  pair and the pair of charged particles associated with the  $V$ .

$\ell^+\ell^-$	$V$					
	$ee$		$\mu\mu$		$\pi\pi$	
	data	Monte Carlo	data	Monte Carlo	data	Monte Carlo
$ee$	11	16.4	5	4.5	5	11.4
$\mu\mu$	8	6.8	2	2.9	8	5.6
$\tau\tau$	4	3.8	2	1.8	5	3.9
totals	23	27.0	9	9.1	18	20.9

18  $\ell^+\ell^-\pi^+\pi^-$  events to be compared with the expected rates of 27.0, 9.1 and 20.9 respectively. Table 3 summarises the classification of these events in both data and Monte Carlo. From fig. 4 and table 3 one sees agreement between data and Monte Carlo both in the overall rates and general shapes of the distributions including the expected enhancement of the  $\ell^+\ell^-\pi^+\pi^-$  rate in the region of the mass of the  $\rho(770)$ . From these results, the 95% confidence level upper limit on a possible absolute anomalous enhancement factor of the  $e^+e^-\mu^+\mu^-$  production rate is 2.3 to be compared with AMY's observed enhancement factor of over three.

The distribution of the cosine of the opening angle between the  $V$  and the closest lepton is presented in fig. 5 for the combined  $\ell^+\ell^-V$  sample. Again the mea-

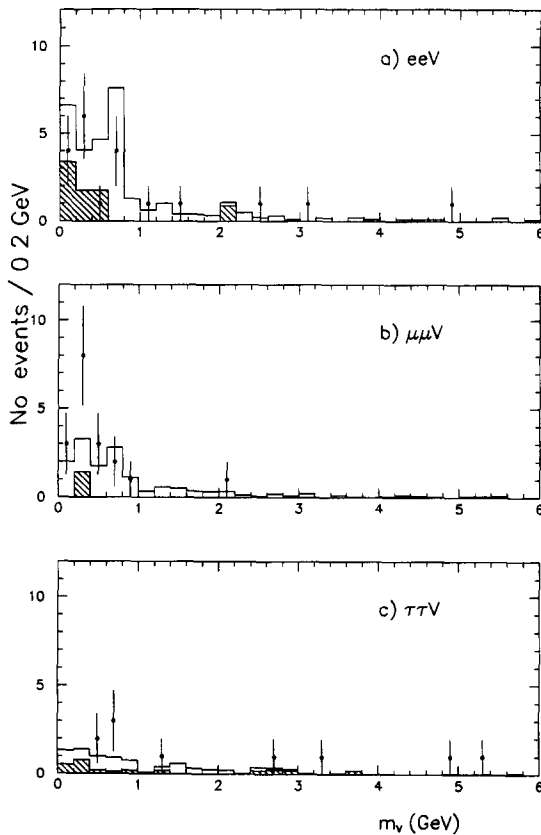


Fig 3 Distributions of the mass of the  $V$  in (a)  $e^+e^-V$  events, (b)  $\mu^+\mu^-V$  events and (c)  $\tau^+\tau^-V$  events after all cuts have been applied. The points represent data and line histograms represent Monte Carlo where the shaded portion represents the contribution from all sources other than  $\ell^+\ell^-V$ .

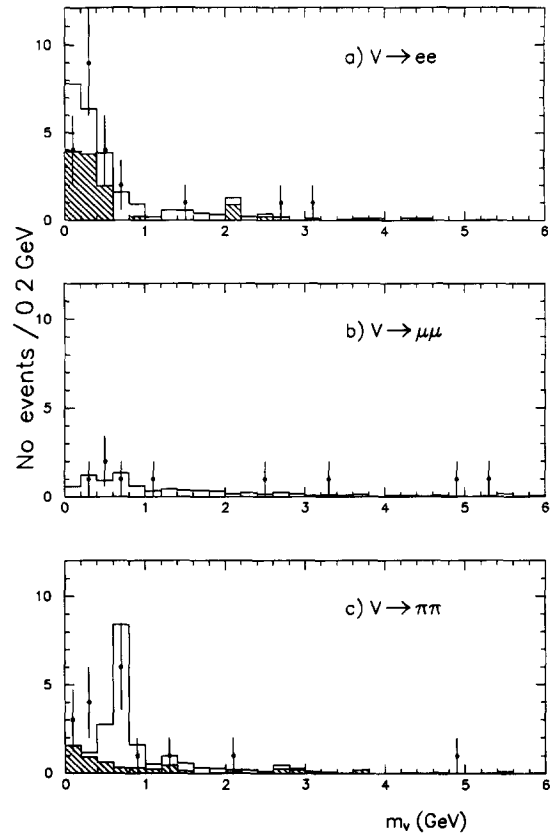


Fig 4 Distributions of the mass of the  $V$  in  $e^+e^-V$ ,  $\mu^+\mu^-V$  and  $\tau^+\tau^-V$  events where (a)  $V \rightarrow ee$ , (b)  $V \rightarrow \mu\mu$ , and (c)  $V \rightarrow \pi\pi$  after all cuts have been applied. The points represent data and line histograms represent Monte Carlo where the shaded portion represents the background contributions. In (a) most of the background is from sources other than  $\ell^+\ell^-V$  while in (c) the background is mainly from  $\ell^+\ell^-\mu^+\mu^-$  events.

sured distribution is fully consistent with the Monte Carlo distribution, and shows a forward peaking characteristic of the bremsstrahlung process.

In conclusion, the agreement between data and Monte Carlo in this analysis, both for the absolute rates and the differential distributions, strongly supports the hypothesis that the selected  $\ell^+\ell^-V$  events come predominantly from the leptonic four-fermion and related hadronic processes and, at the present level of accuracy, are adequately described by the theoretical models [10] discussed above.

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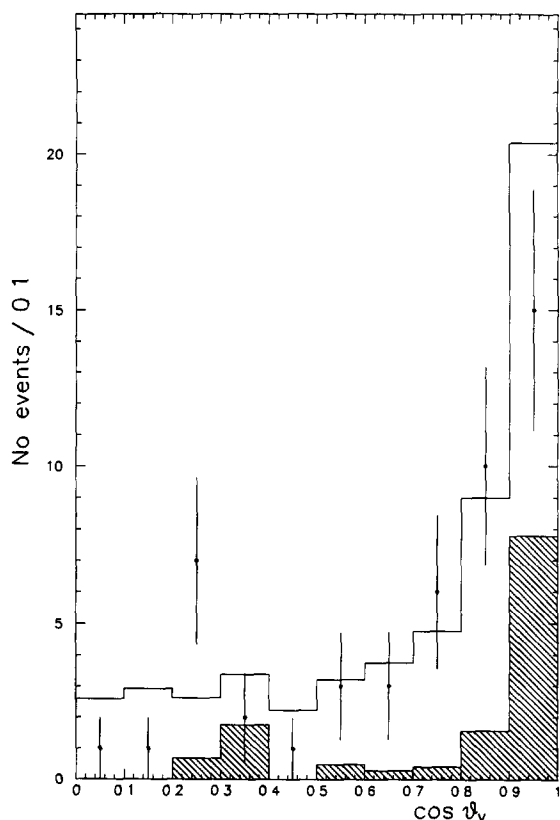


Fig 5 Distribution of the cosine of the opening angle between the  $V$  and the closest lepton, combining all lepton species. The points represent data and line histograms represent Monte Carlo where the contribution from non- $\ell^+\ell^-V$  sources is depicted by the shaded portion of the histogram.

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